Coupled-Mode Analysis of Leaky Waves in Channel Waveguides Consisting of Anisotropic Material

Masahiro Geshiro, *Member, IEEE,* Masashi Hotta, *Member, IEEE,* and Tomotoshi Kameshima

Abstract—**For the first time, a study is made of leaky waves in anisotropic circular channel waveguides consisting of uniaxial crystalline material, in which the optical axis is on the plane which is defined by the propagation axis and one of the transverse coordinates axes. The analysis is based on the theory of coupled modes. Mathematical discretization of the continuum of radiation modes offers satisfactorily accurate solutions of the coupled-mode equations. The characteristics of leakage losses and the field distributions of leaky waves in a $LiNbO_3$ waveguide are discussed on the basis of the numerical results.**

I. Introduction

Many guided-wave devices for optical integrated circuits are fabricated on $LiNbO_3$ or $LiTaO_3$ substrates because of their excellent optical properties and relatively strong electrooptic effects [1]. Therefore, understanding the details of wave propagation in such anisotropic dielectric waveguides is of fundamental interest from the point of view of waveguiding theory as well as practical device planning.

There are two different approaches to waveguide analysis. One is the eigenmode method which seeks the solution of Maxwell's equations that satisfies proper boundary conditions. This is applicable to the analysis of waveguides uniform along the propagation direction. In some two-dimensional waveguides, it is possible to obtain rigorous solutions including leaky-wave solutions [2]–[7]. However, the difficulty in matching all required boundary conditions makes it hard to solve three-dimensional waveguides rigorously even if there is no misalignment between the waveguide and material axes; therefore, only a few cases have been studied [8], [9]. For pure guided modes, some numerical methods have yielded accurate solutions [10]–[12]. However, in order to analyze leaky waves in channel waveguides with axes misalignment, we must try to find complex solutions of complicated vector equations; reliable solutions have not been reported yet.

A coupled-mode analysis may be a powerful approach for such a case. In this approach, the electromagnetic fields in the waveguide under consideration are expanded in terms of normal modes of appropriate waveguides which are similar to the waveguide of interest. The waveguiding properties can be understood from the knowledge of mode-conversion phenomena among the normal modes. Coupled-mode theory is practical because it can describe fields in waveguides that are inhomogeneous along the propagation axis and/or of finite length suitable for integrated-optics devices. When it is possible to obtain normal modes in a certain waveguide structure, any waveguide perturbation can always be treated by means of the coupled-mode analysis [13]–[18].

In the present paper, we analyze the propagation characteristics of leaky waves in a circular channel waveguide consisting of uniaxial crystalline material. The optical axis of the material lies on the plane defined by the propagation axis and one of the transverse coordinate axes. The angle between the optical axis and waveguide axis is referred to as the oblique angle. The leakage of optical waves can be described as a mode-conversion phenomenon between a guided mode and radiation modes. The continuum of radiation modes is discretized for computational convenience in the present analysis. This paper clarifies the basic behavior of leaky waves in an anisotropic channel waveguide for the first time, which deepens our understanding of anisotropic waveguides.

II. Waveguide Structure and Mode Expansion

The waveguide structure under consideration is shown in Fig. 1, together with the coordinate system used for the analysis. Both the core and substrate are assumed to be composed of a uniaxial crystalline material. The z-axis is the propagation direction of the optical waves. It is assumed that the optical axes in the core and substrate regions are parallel and that they make an oblique angle α with the z-axis in the y-z plane. Then, the permittivity tensor in the waveguide coordinate system is expressed as

$$\hat{\varepsilon}_d = \varepsilon_0 \begin{pmatrix} \varepsilon_{xxd} & 0 & 0 \\ 0 & \varepsilon_{yyd} & \varepsilon_{yzd} \\ 0 & \varepsilon_{zyd} & \varepsilon_{zzd} \end{pmatrix} \tag{1}$$

$$\begin{aligned} \varepsilon_{xxd} &= n_{od}^2 \\ \varepsilon_{yyd} &= n_{od}^2 \cos^2\alpha + n_{ed}^2 \sin^2\alpha \\ \varepsilon_{zzd} &= n_{od}^2 \sin^2\alpha + n_{ed}^2 \cos^2\alpha \\ \varepsilon_{yzd} &= \varepsilon_{zyd} = (n_{ed}^2 - n_{od}^2)\sin\alpha\cos\alpha \end{aligned} \tag{2}$$

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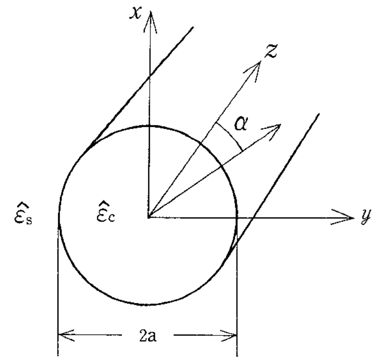

Fig. 1. The waveguide structure and coordinate system.

where n_{od} and n_{ed} are an ordinary and an extraordinary refractive index of the material. The subscript d refers to the core, $d = c$, or the substrate, $d = s$. The free-space permittivity is expressed by ε_0.

Now suppose the waveguide specified by (1) is connected to a circular waveguide, occupying the region $z < 0$, with the same radius and permittivity given by

$$\tilde{\varepsilon}_d = \varepsilon_0 \varepsilon_{xxd} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

Propagating electromagnetic fields in the region $z > 0$ can be expanded in terms of the normal modes of the waveguide specified by (3) as follows:

$$E = \sum_{\nu} a_{g\nu}(z) E_{\nu} + \int_0^{n_{os} k_0} a_r(\rho, z) E(\rho)\, d_P$$

$$H = \sum_{\nu} a_{g\nu}(z) H_{\nu} + \int_0^{n_{os} k_0} a_r(\rho, z) H(\rho) d_P \tag{4}$$

where $a_{g\nu}(z)$ and $a_r(\rho, z)$ are complex amplitude coefficients of the guided and radiation modes, ρ represents the transverse propagation constant of the radiation modes, and k_0 is the free-space wavenumber. For completeness, the upper bound of the integral should be extended to infinity. However, the evanescent modes with spectra in the range of $n_{os}k_0 < \rho < \infty$ which do not carry optical power away from the waveguide are necessary not in order to account for the radiation losses, but to express the fine details of the field close to the waveguide discontinuities. Being interested in the leakage behavior of optical waves in $z > 0$ when a certain guided mode is incident on the interface at $z = 0$, we need only to take normal modes with positive and real propagation constants into consideration in the mode expansions. The effects of reflection at $z = 0$ can be neglected because the difference between the dielectric constants (1) and (3) is very small for most practical cases.

In usual metal-diffused channel waveguides consisting of $LiNbO_3$, for example, the difference of refractive index is very small between the core and substrate regions. Therefore, the LP modes will serve our purpose as an excellent approximation to the rigorous normal modes in the waveguide defined by (3) [19]. Being expressed in rectangular components, the LP modes match the permittivity tensor in mathematical description.

III. Coupled-Mode Analysis

Suppose the waveguide is operated in the single-mode region and either a x-polarized or y-polarized dominant mode is incident on the interface at $z = 0$ from the region $z < 0$. If we neglect the coupling between radiation modes due to the discontinuity, the wave propagation in $z > 0$ can be described by the following coupled-mode equations [13]:

$$\frac{d}{dz} a_g(z) = (-j\beta_g + K_{gg}) a_g(z) + \sum_p \int_0^{n_{os} k_0} K^p_{gr}(\rho) a^p_r(\rho, z)\, d\rho$$

$$\frac{d}{dz} a^p_r(\rho, z) = -j\beta^p_r a^p_r(\rho, z) - K^{p*}_{gr}(\rho) a_g(z) \tag{5}$$

where β_g and β^p_r represent the propagation constants of the guided and radiation modes, respectively. The asterisk indicates complex conjugation and the superscript p refers to the polarization: $p = x$ or y. The coupling coefficients K_{gg} and K_{gr} are expressed as the following overlap integral of the electric field [13]:

$$K_{\mu\nu} = \frac{\omega}{j4W} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E^*_{\mu} \cdot (\hat{\varepsilon}_d - \tilde{\varepsilon}_d) \cdot E_{\nu}\, dx\, dy \tag{6}$$

where ω is the angular frequency and W is the power carried by a normal mode.

It is troublesome to solve (5) analytically in a closed form since the integral with respect to ρ is included; so in the present analysis, we discretize the continuum of radiation modes for computational convenience. Using the increment $\Delta\rho_i$ $(i = 1, 2, \cdots, n)$, the preceding coupled-mode equations become

$$\frac{d}{dz} a_g(z) = (-j\beta_g + K_{gg}) a_g(z) + \sum_p \sum_{i=1}^{n} K^p_{gr}(\rho_i) a^p_{ri}(z)$$

$$\frac{d}{dz} a^p_{ri}(z) = -j\beta^p_{ri} a^p_{ri}(z) - \sqrt{\Delta\rho_i} K^{p*}_{gr}(\rho_i) a_g(z) \tag{7}$$

where ρ_i is assumed to take an intermediate value in the interval $\Delta\rho_i$ for the sake of convenience. The complex amplitude of a discretized radiation mode is defined by

$$a^p_{ri}(z) = \sqrt{\Delta\rho_i}, \quad a^p_r(\rho_i, z) \tag{8}$$

in order that its dimension coincides with that of the guided modes. Readers are referred to [17] for details of the solution to (7).

IV. Numerical Results

The refractive indices in the core and substrate regions are chosen so that $n_{oc} = 2.296$, $n_{os} = 2.286$, $n_{ec} = 2.21$, and

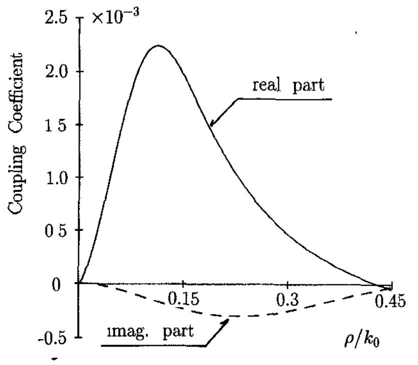

Fig. 2. Coupling coefficients between the y-polarized dominant mode and x-polarized radiation modes as a function of the transverse propagation constant.

$n_{es} = 2.2$, which are the typical values for $LiNbO_3$ channel waveguides in practical use. The waveguide occupying the region $z < 0$ with such indices supports only the dominant mode when $ak_0 < 11$.

Before investigating the details of leaky waves, the accuracy of our solutions should be checked. Fig. 2 shows the coupling coefficients between the y-polarized dominant mode and x-polarized radiation modes when $ak_0 = 8$ and $\alpha = 15°$. The coefficients are normalized relative to k_0. The abscissa represents the normalized transverse propagation constant ρ/k_0. The coupling coefficients for the y-polarized radiation modes have a functional dependence analogous to Fig. 2.

First, let us test the discretization of the ρ-space. The radiation modes with transverse propagation constants in the region $0 < \rho/k_0 < 0.3$ are taken into account and are discretized uniformly. The numerical values for the normalized power of the guided mode are listed in Table I when $k_0z = 500$ and 2000. They are in good agreement up to 4 or 5 significant digits. Judging from these results, the convergence of solutions does not depend critically upon the interval of discretization. However, we have to remark that nonphysical behavior has been observed at large distances, $k_0z > 6000$, when the number of discretized modes is 20. With these results in mind, numerical calculations in the following are carried out with 80-discretized radiation modes, which corresponds the interval of $\Delta\rho_i/k_0 = 0.00375$.

Next, we check the convergence of solutions on the upper bound of the integral ($\rho_{\max}$) with respect to ρ in (5). It has been pointed out in [17] that in actual computations, the upper bound need not be identical with $n_{os}k_0$, but may be far smaller without sacrificing accuracy. The lower bound of the integral should be equal to zero since radiation modes with small values of ρ significantly contribute to the mode coupling even if their coupling coefficients are not large. The numerical values of the solutions are listed in Table II when $k_0z = 500$ and 2000. We can obtain solutions accurate to 4 significant digits when $\rho > 0.28k_0$. Therefore, in the following calculations, we choose $\rho_{\max} = 0.3k_0$, at which the absolute value of the coupling coefficient decreases to one quarter of its peak value.

One of the most important characteristics of the leaky wave is the leakage loss. Fig. 3 illustrates the dependence of power leakage on the angle α for the cases of (a) an incident y-polarized mode and (b) an incident x-polarized mode when $ak_0 = 8$. Only the y-polarized mode exhibits noticeable leaky behavior which can be observed in a narrow region of the angle α. This corresponds to the fact that the TE guided mode becomes leaky in a slab waveguide whose surface is parallel

TABLE I
CONVERGENCE OF SOLUTIONS ON THE NUMBER OF THE DISCRETIZED MODES FOR THE REGION OF $0 < \rho/k_0 < 0.3$

	Power of the Guided Wave	
Number of Discretized Modes	$k_0z = 500$	$k_0z = 2000$
20	0.486338	0.033799
40	0.486334	0.033783
60	0.486339	0.033782
80	0.486338	0.033782
120	0.486338	0.033782

TABLE II
CONVERGENCE OF SOLUTIONS ON THE UPPERD BOUND OF THE INTEGRAL IN (5), WHERE $\Delta\rho_i/k_0 = 0.00375$

	Power of the Guided Wave	
$\rho_{\max}/k_0$ ($\rho_{\max}/\Delta\rho_i$)	$k_0z = 500$	$k_0z = 2000$
0.11 (30)	0.729722	0.035042
0.17 (45)	0.526194	0.027664
0.22 (60)	0.485945	0.033575
0.28 (75)	0.486384	0.033751
0.45 (120)	0.486233	0.033840

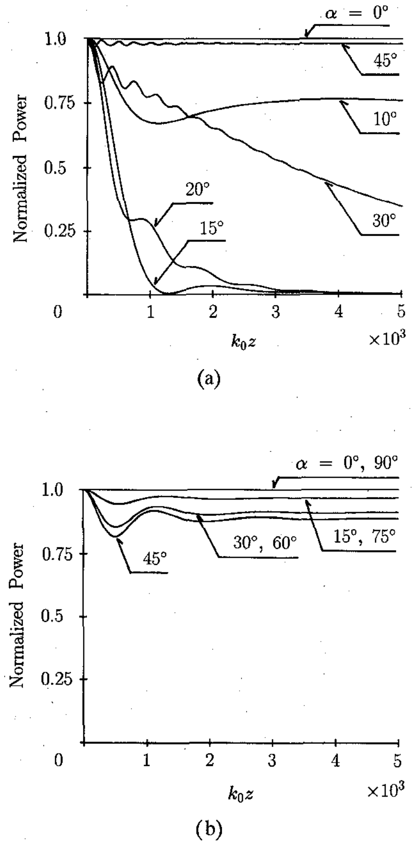

Fig. 3. Power change of the incident mode as a function of the propagation distance with the oblique angle as a parameter: (a) the y-polarized mode incidence and (b) the x-polarized mode incidence.

to the y-z plane in the coordinate system in Fig. 1[7]. Leaky waves do not exist in a slab waveguide with the optical axis in the y-z plane whose surface is parallel to the x-z plane [6]. In the circular channel waveguides which are considered in the present paper, however, the x-polarized mode would become leaky instead if the optical axis is in the x-z plane. This fact can be easily understood from symmetry considerations of the structure. Although the actual sections of practical channel waveguides are somewhat different from Fig. 1, the physical principles are qualitatively the same. Therefore, one of the orthogonally polarized modes becomes leaky in general cylindrical waveguides composed of uniaxial whose optical axis lies on the plane which is defined by the propagation axis and one of the transverse coordinate axes. An absolute single-mode single-polarized operation may be possible by means of these structures.

Another important property is the filed distribution. Fig. 4 shows the power density of the leaky wave observed in the $x = 0$ and $y = 0$ planes along the propagation axis when $ak_0 = 8$ and $\alpha = 15°$. The bird's eye views of the power density for the radiated field on the cross sections at $k_0z = 250$, 750, and 1500 are illustrated in Fig. 5. It is recognized from these figures that the optical wave leaks its power mainly in the plane which contains the optical axis. This behavior is quite a distinct difference from that of a slab waveguide.

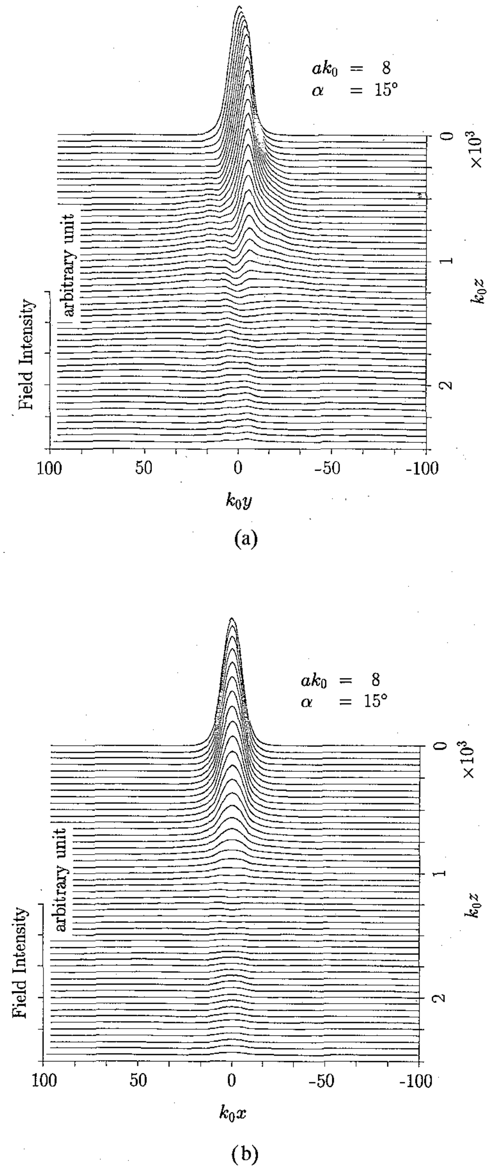

Fig. 4. Power density of the leaky wave observed (a) in the $x = 0$ plane and (b) in the $y = 0$ plane.

V. Conclusion

We have investigated the wave propagation in a circular channel waveguide consisting of uniaxial crystalline material. The optical axis of the material lies on the plane which is defined by the propagation axis and one of the transverse coordinate axes. The analytical method is based on the coupled-mode theory, where the coupling between radiation modes is neglected. The continuum of radiation modes is discreted for the convenience of numerical integration. Numerical data are presented for waveguides composed of $LiNbO_3$, a representative uniaxial crystalline material used for optical devices. The leakage behavior for oblique propagation is discussed with the oblique angle as a parameter. An absolute single-mode single-polarized operation is suggested. Field distributions of leaky waves are also presented which exhibit notable differences from those of leaky waves in slab waveguides.

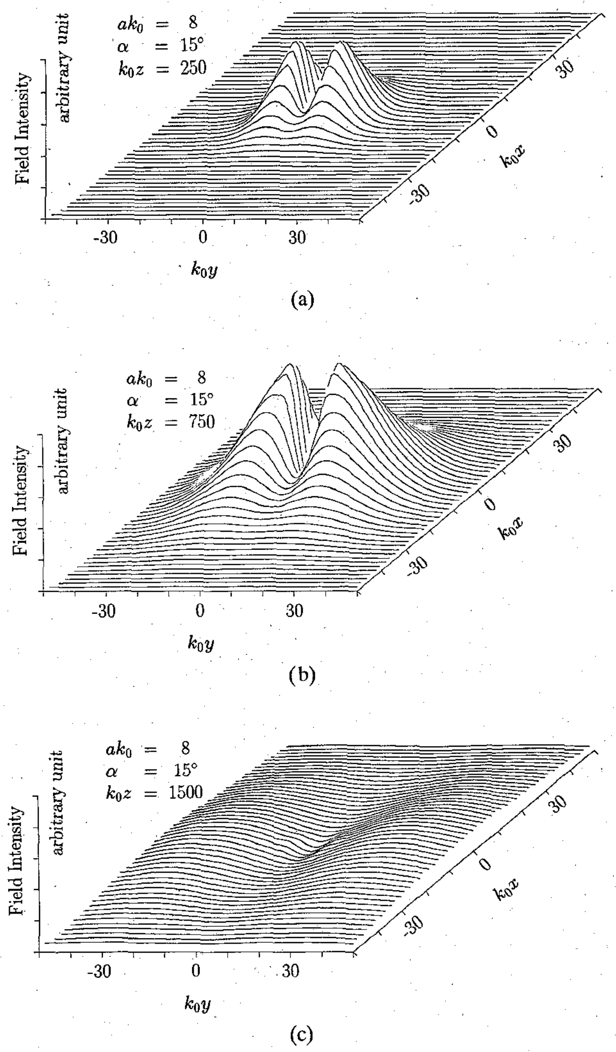

Fig. 5. Bird's eye views of the power density for the radiated field on the cross sections at (a) $k_0z = 250$, (b) $k_0z = 750$, and (c) $k_0z = 1500$.

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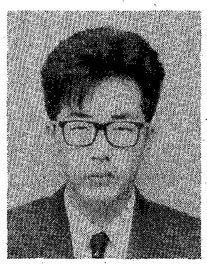

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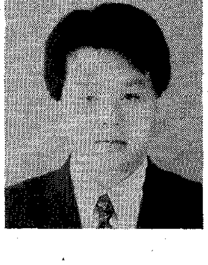

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